## GATE EE

2012

## Q. 1-Q. 25 carry one mark each.

MCQ 1.1 Two independent random variables $X$ and $Y$ are uniformly distributed in the interval $[-1,1]$. The probability that $\max [X, Y]$ is less than $1 / 2$ is
(A) $3 / 4$
(B) $9 / 16$
(C) $1 / 4$
(D) $2 / 3$

SOL 1.1 Option (B) is correct.
Probability density function of uniformly distributed variables $X$ and $Y$ is shown


Since $X$ and $Y$ are independent.

$$
\begin{aligned}
P\left\{[\max (x, y)]<\frac{1}{2}\right\} & =P\left(X<\frac{1}{2}\right) P\left(Y<\frac{1}{2}\right) \\
P\left(X<\frac{1}{2}\right) & =\text { shaded area }=\frac{3}{4}
\end{aligned}
$$

Similarly for $Y: \quad P\left(Y<\frac{1}{2}\right)=\frac{3}{4}$
So

$$
P\left\{[\max (x, y)]<\frac{1}{2}\right\}=\frac{3}{4} \times \frac{3}{4}=\frac{9}{16}
$$

## Alternate method:



From the given data since random variables $X$ and $Y$ lies in the interval $[-1,1]$ as from the figure $X, Y$ lies in the region of the square $A B C D$.
Probability for $\max [X, Y]<1 / 2$ : The points for $\max [X, Y]<1 / 2$ will be inside the region of square $A E F G$.

So,

$$
P\left\{\max [X, Y]<\frac{1}{2}\right\}=\frac{\text { Area of } \square A E F G}{\text { Area of square } A B C D}
$$

MCQ 1.2 If $x=\sqrt{-1}$, then the value of $x^{x}$ is (A) $e^{-\pi / 2}$
(C) $x$
(D) 1

SOL 1.2 Option (A) is correct.

$$
x=\sqrt{-1}=i=\cos \frac{\pi}{2}+i \sin \frac{\pi}{2}
$$

So,

$$
\begin{aligned}
x & =e^{i \frac{\pi}{2}} \\
x^{x} & =\left(e^{i \frac{\pi}{2}}\right)^{x} \Rightarrow\left(e^{i \frac{\pi}{2}}\right)^{i} \\
& =e^{-\frac{\pi}{2}}
\end{aligned}
$$

MCQ 1.3 Given $f(z)=\frac{1}{z+1}-\frac{2}{z+3}$.
If $C$ is a counter clockwise path in the $z$-plane such that $|z+1|=1$, the value of $\frac{1}{2 \pi j} \oint_{C} f(z) d z$ is
(A) -2
(B) -1
(C) 1
(D) 2

SOL 1.3 Option (C) is correct.

$$
f(z)=\frac{1}{z+1}-\frac{2}{z+3}
$$

$$
\frac{1}{2 \pi j} \oint_{C} f(z) d z=\underset{\text { sum of the residues of the poles which lie inside the }}{\text { given closed region. }}
$$

$$
C \Rightarrow|z+1|=1
$$

Only pole $z=-1$ inside the circle, so residue at $z=-1$ is.

$$
\begin{aligned}
f(z) & =\frac{-z+1}{(z+1)(z+3)} \\
& =\lim _{z \rightarrow-1} \frac{(z+1)(-z+1)}{(z+1)(z+3)}=\frac{2}{2}=1
\end{aligned}
$$

So

$$
\frac{1}{2 \pi j} \oint_{C} f(z) d z=1
$$

MCQ 1.4 In the circuit shown below, the current through the inductor is

(A) $\frac{2}{1+j} \mathrm{~A}$
(B) $\frac{-1}{1+j} \mathrm{~A}$
(C) $\frac{1}{1+j} \mathrm{~A}$
(D) 0 A

SOL 1.4 Option (C) is correct.


Applying nodal analysis at top node.

Current

$$
\begin{aligned}
\frac{\boldsymbol{V}_{1}+1 \angle 0^{\circ}}{1}+\frac{\boldsymbol{V}_{1}+1 \angle 0^{\circ}}{j 1} & =1 \angle 0^{\circ} \\
\boldsymbol{V}_{1}(j 1+1)+j 1+1 \angle 0^{\circ} & =j 1 \\
\boldsymbol{V}_{1} & =\frac{-1}{1+j 1}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{C}_{1} & =\frac{\boldsymbol{V}_{1}+1 / 0^{\circ}}{j 1}=\frac{-\frac{1}{1+j}+1}{j 1} \\
& =\frac{\boldsymbol{\sigma}_{j}}{(1+j) j}=\frac{1}{1+j} \mathrm{~A}
\end{aligned}
$$

MCQ 1.5 The impedance looking into nodes 1 and 2 in the given circuit is

(A) $50 \Omega$
(B) $100 \Omega$
(C) $5 \mathrm{k} \Omega$
(D) $10.1 \mathrm{k} \Omega$

SOL 1.5 Option (A) is correct.
We put a test source between terminal 1, 2 to obtain equivalent impedance
$9 \mathrm{k} \Omega$


$$
Z_{T h}=\frac{V_{\text {test }}}{I_{\text {test }}}
$$

By applying KCL at top right node

$$
\begin{align*}
\frac{V_{\text {test }}}{9 \mathrm{k}+1 \mathrm{k}}+\frac{V_{\text {test }}}{100}-99 I_{b} & =I_{\text {test }} \\
\frac{V_{\text {test }}}{10 \mathrm{k}}+\frac{V_{\text {test }}}{100}-99 I_{b} & =I_{\text {test }} \tag{i}
\end{align*}
$$

But

$$
I_{b}=-\frac{V_{\text {test }}}{9 k+1 k}=-\frac{V_{\text {test }}}{10 \mathrm{k}}
$$

Substituting $I_{b}$ into equation (i), we have

$$
\begin{aligned}
\frac{V_{\text {test }}}{10 \mathrm{k}}+\frac{V_{\text {test }}}{100}+\frac{99 V_{\text {test }}}{10 \mathrm{k}} & =I_{\text {test }} \\
\frac{100 V_{\text {test }}}{10 \times 10^{3}}+\frac{V_{\text {test }}}{100} & =I_{\text {test }} \\
\frac{2 V_{\text {test }}}{100} & =I_{\text {test }} \\
Z_{\text {Th }} & =\frac{V_{\text {test }}}{I_{\text {test }}}=50 \Omega
\end{aligned}
$$

MCQ 1.6 A system with transfer function

$$
G(s)=\frac{\left(s^{2}+9\right)(s+2)}{(s+1)(s+3)(s+4)}
$$

is excited by $\sin (\omega t)$. The steady-state output of the system is zero at
(A) $\omega=1 \mathrm{rad} / \mathrm{s}$
(B) $\omega=2 \mathrm{rad} / \mathrm{s}$
(C) $\omega=3 \mathrm{rad} / \mathrm{s}$
(D) $\omega=4 \mathrm{rad} / \mathrm{s}$

SOL 1.6 Option (C) is correct.

$$
\begin{aligned}
G(s) & =\frac{\left(s^{2}+9\right)(s+2)}{(s+1)(s+3)(s+4)} \\
G(j \omega) & =\frac{\left(-\omega^{2}+9\right)(j \omega+2)}{(j \omega+1)(j \omega+3)(j \omega+4)}
\end{aligned}
$$

The steady state output will be zero if

$$
\begin{aligned}
|G(j \omega)| & =0 \\
-\omega^{2}+9 & =0 \\
\omega & =3 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

MCQ 1.7 In the sum of products function $f(X, Y, Z)=\sum(2,3,4,5)$, the prime implicants are
(A) $\bar{X} Y, X \bar{Y}$
(B) $\bar{X} Y, X \bar{Y} \bar{Z}, X \bar{Y} Z$
(C) $\bar{X} Y \bar{Z}, \bar{X} Y Z, X \bar{Y}$
(D) $\bar{X} Y \bar{Z}, \bar{X} Y Z, X \bar{Y} \bar{Z}, X \bar{Y} Z$

SOL 1.7 Option (A) is correct.
Prime implicants are the terms that we get by solving K-map

| $Y Z$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 |
| 1 | 1 | 1 |  |  |
| 0 |  |  | 1 | 1 |

$$
F=\underbrace{X \bar{Y}+\bar{X} Y}_{\text {prime implicants }}
$$

MCQ 1.8 If $x[n]=(1 / 3)^{|n|}-(1 / 2)^{n} u[n]$, then the region of convergence (ROC) of its $z$ -transform in the $z$-plane will be
(A) $\frac{1}{3}<|z|<3$
(B) $\frac{1}{3}<|z|<\frac{1}{2}$
(C) $\frac{1}{2}<|z|<3$

SOL 1.8 Option (C) is correct.

$$
\begin{aligned}
& x[n]=\left(\frac{1}{3}\right)^{|n|}-\left(\frac{1}{2}\right)^{n} u[n] \\
& x[n]=\left(\frac{1}{3}\right)^{n} u[n]+\left(\frac{1}{3}\right)^{-n} u[-n-1]-\left(\frac{1}{2}\right)^{n} u(n)
\end{aligned}
$$

Taking $z$-transform

$$
\begin{aligned}
& X[z]=\sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{n} z^{-n} u[n]+\sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{-n} z^{-n} u[-n-1] \\
& -\sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^{n} z^{-n} u[n] \\
& =\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n} z^{-n}+\sum_{n=-\infty}^{-1}\left(\frac{1}{3}\right)^{-n} z^{-n}-\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n} z^{-n} \\
& =\underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{3 z}\right)^{n}}_{\mathrm{I}}+\underbrace{\sum_{m=1}^{\infty}\left(\frac{1}{3} z\right)^{m}}_{\mathrm{II}}-\underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2 z}\right)^{n}}_{\mathrm{III}} \quad \quad \text { Taking } m=-n
\end{aligned}
$$

Series I converges if $\left|\frac{1}{3 z}\right|<1$ or $|z|>\frac{1}{3}$
Series II converges if $\left|\frac{1}{3} z\right|<1$ or $|z|<3$
Series III converges if $\left|\frac{1}{2 z}\right|<1$ or $|z|>\frac{1}{2}$

Region of convergence of $X(z)$ will be intersection of above three
So, $\quad \operatorname{ROC}: \frac{1}{2}<|z|<3$
MCQ 1.9 The bus admittance matrix of a three-bus three-line system is

$$
Y=j\left[\begin{array}{rrr}
-13 & 10 & 5 \\
10 & -18 & 10 \\
5 & 10 & -13
\end{array}\right]
$$

If each transmission line between the two buses is represented by an equivalent $\pi$ -network, the magnitude of the shunt susceptance of the line connecting bus 1 and 2 is
(A) 4
(B) 2
(C) 1
(D) 0

SOL 1.9 Option (B) is correct.
For bus admittance matrix,

$$
\begin{aligned}
Y_{11}+\left(Y_{12}+y_{\text {line }}\right)+Y_{13} & =0 \\
-j 13+\left(j 10+y_{\text {line }}\right)+j 5 & =0 \\
y_{\text {line }} & =-j 2
\end{aligned}
$$

Magnitude of susceptance is +2
MCQ 1.10 The slip of an induction motor normally does not depend on
(A) rotor speed
(C) shaft torque

## (B) synchronous speed —(D) core-loss component

SOL 1.10 Option (D) is correct.
Slip is given as

$$
S=\frac{n_{s}-n}{n_{s}}
$$

where,

$$
\begin{aligned}
n_{s} & =\text { synchronous speed } \\
n & =\text { rotor speed }
\end{aligned}
$$

Thus, slip depend on synchronous speed and the rotor speed. Also, torque increases with increasing slip up to a maximum value and then decreases. Slip does not depend on core/loss component.

MCQ 1.11 A two-phase load draws the following phase currents : $i_{1}(t)=I_{m} \sin \left(\omega t-\phi_{1}\right)$, $i_{2}(t)=I_{m} \cos \left(\omega t-\phi_{2}\right)$. These currents are balanced if $\phi_{1}$ is equal to.
(A) $-\phi_{2}$
(B) $\phi_{2}$
(C) $\left(\pi / 2-\phi_{2}\right)$
(D) $\left(\pi / 2+\phi_{2}\right)$

SOL 1.11 Option (A) is correct.

$$
\begin{aligned}
& i_{1}(t)=I_{m} \sin \left(\omega t-\phi_{1}\right) \\
& i_{2}(t)=I_{m} \cos \left(\omega t-\phi_{2}\right)
\end{aligned}
$$

We know that,

$$
\cos \left(\theta-90^{\circ}\right)=\sin \theta
$$

So, $i_{1}(t)$ can be written as

$$
\begin{aligned}
& i_{1}(t)=I_{m} \cos \left(\omega t-\phi_{1}-90^{\circ}\right) \\
& i_{2}(t)=I_{m} \cos \left(\omega t-\phi_{2}\right)
\end{aligned}
$$

Now, in phasor form

$$
\begin{aligned}
& \boldsymbol{I}_{1}=I_{m} \angle \phi_{1}+90^{\circ} \\
& \boldsymbol{I}_{2}=I_{m} \angle \phi_{2}
\end{aligned}
$$

Current are balanced if

$$
\boldsymbol{I}_{1}+\boldsymbol{I}_{2}=0
$$

$$
I_{m} \angle \phi_{1}+90^{\circ}+I_{m} \angle \phi_{2}=0
$$

$$
I_{m} \cos \left(\phi_{1}+90^{\circ}\right)+j I_{m} \sin \left(\phi_{1}+90^{\circ}\right)+\cos \phi_{2}+j \sin \phi_{2}=0
$$

$$
I_{m}\left[\cos \left(\phi_{1}+90^{\circ}\right)+j \sin \left(\phi_{1}+90^{\circ}\right)\right]+I_{m}\left[\cos \phi_{2}+j \sin \phi_{2}\right]=0
$$

$$
I_{m}\left[\cos \left(\phi_{1}+90^{\circ}\right)+\cos \phi_{2}\right]+j I_{m}\left[\sin \phi_{2}+\sin \left(\phi_{1}+90^{\circ}\right)\right]=0
$$

$$
\begin{aligned}
\cos \left(\phi_{1}+90^{\circ}\right)+\cos \phi_{2} & =0 \\
\cos \left(\phi_{1}+90^{\circ}\right) & =-\cos \phi_{2}=\cos \left(\pi+\phi_{2}\right) \\
\phi_{1}+90^{\circ} & =\pi+\phi_{2}
\end{aligned}
$$

or,
MCQ 1.12 A periodic voltage waveform observed on an oscilloscope across a load is shown. A permanent magnet moving coil (PMMC) meter connected across the same load reads

(A) 4 V
(B) 5 V
(C) 8 V
(D) 10 V

SOL 1.12 Option (A) is correct.
PMMC instrument reads average (dc) value.

$$
\begin{aligned}
V_{a v g} & =\frac{1}{T} \int_{0}^{T} v(t) d t \\
& =\frac{1}{20 \times 10^{-3}} \int_{0}^{20} v(t) d t
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{1}{20}\left[\int_{0}^{10} t d t+\int_{10}^{12}(-5) d t+\int_{12}^{20} 5 d t\right] \\
& =\frac{1}{20}\left(\left[\frac{t^{2}}{2}\right]_{0}^{10}-5[t]_{10}^{12}+5[t]_{12}^{20}\right) \\
& =\frac{1}{20}[50-5(2)+5(8)] \\
& =\frac{80}{20}=4 \mathrm{~V}
\end{aligned}
$$

MCQ 1.13 The bridge method commonly used for finding mutual inductance is
(A) Heaviside Campbell bridge
(B) Schering bridge
(C) De Sauty bridge
(D) Wien bridge

SOL 1.13 Option (A) is correct.
Heaviside mutual inductance bridge measures mutual inductance is terms of a known self-inductance.

MCQ 1.14 With initial condition $x(1)=0.5$, the solution of the differential equation $t \frac{d x}{d t}+x=t$, is
(A) $x=t-\frac{1}{2}$
(C) $x=\frac{t^{2}}{2}$

SOL 1.14 Option (D) is correct.
(B) $x=t^{2}-\frac{1}{2}$

$$
\begin{aligned}
t \frac{d x}{d t}+x & =t \\
\frac{d x}{d t}+\frac{x}{t} & =1 \\
\frac{d x}{d t}+P x & =Q(\text { General form })
\end{aligned}
$$

Integrating factor,

$$
I F=e^{\int^{p+U}}=e^{f_{t^{\prime}}}=e^{\ln t}=t
$$

Solution has the form

$$
\begin{aligned}
x \times I F & =\int(Q \times I F) d t+C \\
x \times t & =\int(1)(t) d t+C \\
x t & =\frac{t^{2}}{2}+C
\end{aligned}
$$

Taking the initial condition

$$
\begin{aligned}
x(1) & =0.5 \\
0.5 & =\frac{1}{2}+C \\
C & =0
\end{aligned}
$$

So,

$$
x t=\frac{t^{2}}{2} \Rightarrow x=\frac{t}{2}
$$

MCQ 1.15 The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^{2}+s+1}$. The unilateral Laplace transform of $t f(t)$ is
(A) $-\frac{s}{\left(s^{2}+s+1\right)^{2}}$
(B) $-\frac{2 s+1}{\left(s^{2}+s+1\right)^{2}}$
(C) $\frac{s}{\left(s^{2}+s+1\right)^{2}}$
(D) $\frac{2 s+1}{\left(s^{2}+s+1\right)^{2}}$
sOL 1.15 Option (D) is correct.
Using $s$-domain differentiation property of Laplace transform.
If

$$
\begin{aligned}
& f(t) \stackrel{L}{\longleftrightarrow} F(s) \\
& t f(t) \stackrel{\mathcal{L}}{\longleftrightarrow}-\frac{d F(s)}{d s}
\end{aligned}
$$

So,

$$
\begin{aligned}
\mathcal{L}[t f(t)] & =\frac{-d}{d s}\left[\frac{1}{s^{2}+s+1}\right] \\
& =\frac{2 s+1}{\left(s^{2}+s+1\right)^{2}}
\end{aligned}
$$

MCQ 1.16 The average power delivered to an impedance $(4-j 3) \Omega$ by a current $5 \cos (100 \pi t+100) \mathrm{A}$ is
(A) 44.2 W
(C) 62.5 W

sOL 1.16 Option (B) is correct.
In phasor form

$$
\begin{aligned}
Z & =4-j 3 \\
Z & =5 /-36.86^{\circ} \Omega \\
I & =5 / 100^{\circ} \mathrm{A}
\end{aligned}
$$

Average power delivered.

$$
\begin{aligned}
P_{\text {avg }} & =\frac{1}{2}|\boldsymbol{I}|^{2} Z \cos \theta \\
& =\frac{1}{2} \times 25 \times 5 \cos 36.86^{\circ} \\
& =50 \mathrm{~W}
\end{aligned}
$$

## Alternate method:

$$
\begin{aligned}
Z & =(4-j 3) \Omega \\
I & =5 \cos (100 \pi t+100) \mathrm{A} \\
P_{\text {avg }} & =\frac{1}{2} \operatorname{Re}\left\{|I|^{2} Z\right\} \\
& =\frac{1}{2} \times \operatorname{Re}\left\{(5)^{2} \times(4-j 3)\right\}
\end{aligned}
$$

$$
=\frac{1}{2} \times 100=50 \mathrm{~W}
$$

MCQ 1.17 In the following figure, $C_{1}$ and $C_{2}$ are ideal capacitors. $C_{1}$ has been charged to 12 V before the ideal switch $S$ is closed at $t=0$. The current $i(t)$ for all $t$ is

(A) zero
(B) a step function
(C) an exponentially decaying function
(D) an impulse function

SOL 1.17 Option (D) is correct.
The $s$-domain equivalent circuit is shown as below.

$v_{C}(0)=12 \mathrm{~V}$

Taking inverse Laplace transform for the current in time domain,

$$
i(t)=12 C_{e q} \delta(t)
$$

MCQ 1.18 The $i-v$ characteristics of the diode in the circuit given below are

$$
i= \begin{cases}\frac{v-0.7}{500} \mathrm{~A}, & v \geq 0.7 \mathrm{~V} \\ 0 \mathrm{~A} & v<0.7 \mathrm{~V}\end{cases}
$$



The current in the circuit is
(A) 10 mA
(B) 9.3 mA
(C) 6.67 mA
(D) 6.2 mA

SOL 1.18 Option (D) is correct.
Let $v>0.7 \mathrm{~V}$ and diode is forward biased. By applying Kirchoff's voltage law

$$
\begin{aligned}
10-i \times 1 \mathrm{k}-v & =0 \\
10-\left[\frac{v-0.7}{500}\right](1000)-v & =0 \\
10-(v-0.7) \times 2-v & =0 \\
10-3 v+1.4 & =0 \\
v & =\frac{11.4}{3}=3.8 \mathrm{~V}>0.7 \quad \text { (Assumption is true) }
\end{aligned}
$$

So,

$$
i=\frac{v-0.7}{500}=\frac{3.8-0.7}{500}=6.2 \mathrm{~mA}
$$

MCQ 1.19 The output $Y$ of a 2-bit comparator is logic 1 whenever the 2-bit input $A$ is greater than the 2 -bit input $B$. The number of combinations for which the output is logic 1 , is
(A) 4
(B) 6
(C) 8
(D) 10

SOL 1.19 Option (B) is correct.


$$
\begin{aligned}
& Y=1, \text { when } A>B \\
& A=a_{1} a_{0}, B=b_{1} b_{0}
\end{aligned}
$$

| $a_{1}$ | $a_{0}$ | $b_{1}$ | $b_{0}$ | $Y$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |

Total combination $=6$
MCQ 1.20 Consider the given circuit


In this circuit, the race around
(A) does not occur
(B) occur when $C L K=0$
(C) occur when $C L K=1$ and $A=B=1$
(D) occur when $C L K=1$ and $A=B=0$

SOL 1.20 Option (A) is correct.
The given circuit is


Condition for the race-around
It occurs when the output of the circuit $\left(Y_{1}, Y_{2}\right)$ oscillates between ' 0 ' and ' 1 ' checking it from the options.

1. Option (A): When $C L K=0$

Output of the NAND gate will be $A_{1}=B_{1}=\overline{0}=1$. Due to these input to the next NAND gate, $Y_{2}=\overline{Y_{1} \cdot 1}=\overline{Y_{1}}$ and $Y_{1}=\overline{Y_{2} \cdot 1}=\overline{Y_{2}}$.
If $Y_{1}=0, \quad Y_{2}=\bar{Y}_{1}=1$ and it will remain the same and doesn't oscillate.
If $Y_{2}=0, \quad Y_{1}=\overline{Y_{2}}=1$ and it will also remain the same for the clock period. So, it won't oscillate for $C L K=0$.
So, here race around doesn't occur for the condition $C L K=0$.
2. Option (C): When $C L K=1, A=B=1$

$$
A_{1}=B_{1}=0 \text { and so } Y_{1}=Y_{2}=1
$$

And it will remain same for the clock period. So race around doesn't occur for the condition.
3. Option (D): When $C L K=1, A=B=0$

So, $\quad A_{1}=B_{1}=1$
And again as described for Option (B) race around doesn't occur for the condition. So, Option (A) will be correct.

MCQ 1.21 The figure shows a two-generator system applying a load of $P_{D}=40 \mathrm{MW}$, connected at bus 2 .


The fuel cost of generators $G_{1}$ and $G_{2}$ are :
$C_{1}\left(P_{G 1}\right)=10000 \mathrm{Rs} / \mathrm{MWh}$ and $C_{2}\left(P_{G 2}\right)=12500 \mathrm{Rs} / \mathrm{MWh}$ and the loss in the line is $P_{\text {loss }(p u)}=0.5 P_{G 1(p u)}^{2}$, where the loss coefficient is specified in pu on a 100 MVA base.
The most economic power generation schedule in MW is
(A) $P_{G 1}=20, P_{G 2}=22$
(B) $P_{G 1}=22, P_{G 2}=20$
(C) $P_{G 1}=20, P_{G 2}=20$
(D) $P_{G 1}=0, P_{G 2}=40$

SOL 1.21 Option (A) is correct.
Let penalty factor of plant $G$, is $L_{1}$ given as

So,

$$
\begin{aligned}
& L_{1}=\frac{1}{1-\frac{\partial P_{L}}{\partial P_{G_{1}}}} \\
& P_{L}=0.5 P_{G_{1}}^{2} \\
& \frac{\partial P_{L}}{\partial P_{G_{2}}}=0.5\left(2 P_{G_{1}}\right)=P_{G_{1}} \\
& L_{1}=\frac{1}{1-P_{G_{2}}} \\
& \mathrm{t} G_{2} \text { is }
\end{aligned}
$$

$$
L_{2}=\frac{1}{1-\frac{\partial P_{L}}{\partial P_{G_{2}}}}=1
$$

$$
\left(\because \frac{\partial P_{L}}{\partial P_{G_{2}}}=0\right)
$$

For economic power generation

$$
C_{1} \times L_{1}=C_{2} \times L_{2}
$$

where $C_{1}$ and $C_{2}$ are the incremental fuel cost of plant $G_{1}$ and $G_{2}$.
So,

$$
\begin{aligned}
(10000)\left(\frac{1}{1-P_{G_{2}}}\right) & =12500 \times 1 \\
\frac{4}{5} & =1-P_{G_{2}} \\
P_{G_{2}} & =\frac{1}{5} \mathrm{pu}
\end{aligned}
$$

It is an 100 MVA , so

Loss

$$
P_{G_{2}}=\frac{1}{5} \times 100=20 \mathrm{MW}
$$

$$
P_{L}=0.5\left(\frac{1}{5}\right)^{2}=\frac{1}{50} \mathrm{pu}
$$

or

$$
P_{L}=\frac{1}{50} \times 100=2 \mathrm{MW}
$$

Total power,

$$
\begin{aligned}
P_{L} & =P_{G_{1}}+P_{G_{2}}-P_{L} \\
40 & =20+P_{2}-2 \\
P_{G_{2}} & =22 \mathrm{MW}
\end{aligned}
$$

MCQ 1.22 The sequence components of the fault current are as follows : $I_{\text {positive }}=j 1.5 \mathrm{pu}$, $I_{\text {negative }}=-j 0.5 \mathrm{pu}, I_{\text {zero }}=-j 1 \mathrm{pu}$. The type of fault in the system is
(A) $L G$
(B) $L L$
(C) $L L G$
(D) $L L L G$

SOL 1.22 Option (C) is correct.
For double line-to-ground $(L L G)$ fault, relation between sequence current is

$$
I_{\text {positive }}=-\left(I_{\text {negative }}+I_{\text {zero }}\right)
$$

Gives values satisfy this relation, therefore the type of fault is $L L G$.
MCQ 1.23 A half-controlled single-phase bridge rectifier is supplying an $R$ - $L$ load. It is operated at a firing angle $\alpha$ and the load current is continuous. The fraction of cycle that the freewheeling diode conducts is
(A) $1 / 2$
(B) $(1-\alpha / \pi)$
(C) $\alpha / 2 \pi$
(D) $\alpha / \pi$

SOL 1.23 Option (D) is correct
The circuit of a single-phase half controlled bridge rectifier with $R L$ load and free wheel diode is shown as below.


The voltage current wave forms are shown in figure below.


We note that, for continuous load current, the flywheel diode conducts from $\pi$ to $\pi+\alpha$ in a cycle. Thus, fraction of cycle that freewheel diode conducts is $\alpha / \pi$.
Thus fraction of cycle that freewheel diode conducts is $\alpha / \pi$.
MCQ 1.24 The typical ratio of latching current to holding current in a 20 A thyristor is
(A) 5.0
(B) 2.0
(C) 1.0
(D) 0.5

SOL 1.24 Option (B) is correct.
The latching current is higher than the holding current. Usually, latching current is taken two to three times the holding currents.

MCQ 1.25 For the circuit shown in the figure, the voltage and current expressions are
$v(t)=E_{1} \sin (\omega t)+E_{3} \sin (3 \omega t)$ and
$i(t)=I_{1} \sin \left(\omega t-\phi_{1}\right)+I_{3} \sin \left(3 \omega t-\phi_{3}\right)+I_{5} \sin (5 \omega t)$
The average power measured by the wattmeter is

(A) $\frac{1}{2} E_{1} I_{1} \cos \phi_{1}$
(B) $\frac{1}{2}\left[E_{1} I_{1} \cos \phi_{1}+E_{1} I_{3} \cos \phi_{3}+E_{1} I_{5}\right]$
(C) $\frac{1}{2}\left[E_{1} I_{1} \cos \phi_{1}+E_{3} I_{3} \cos \phi_{3}\right]$
(D) $\frac{1}{2}\left[E_{1} I_{1} \cos \phi_{1}+E_{3} I_{1} \cos \phi_{1}\right]$

SOL 1.25 Option (C) is correct.
Let $\omega t=\theta$, we have instaneous voltage and current as follows.

$$
\begin{aligned}
v(t) & =E_{1} \sin \theta+E_{3} \sin 3 \theta \\
i(t) & =I_{1} \sin \left(\theta-\phi_{1}\right)+I_{3} \sin \left(3 \theta-\phi_{3}\right)+I_{5} \sin (5 \theta)
\end{aligned}
$$

We know that wattmeter reads the average power, which is gives as

$$
\begin{equation*}
P=\frac{1}{2 \pi} \int_{0}^{2 \pi} v(t) i(t) d \theta \tag{i}
\end{equation*}
$$

We can solve this integration using following results.
(i) $\frac{1}{2 \pi} \int_{0}^{2 \pi} A \sin (\theta+\alpha) \cdot B \sin (\theta+\beta) d \theta=\frac{1}{2} A B \cos (\alpha-\beta)$
(ii) $\frac{1}{2 \pi} \int_{0}^{2 \pi} A \sin (\theta+\alpha) \cdot B \cos (\theta+\alpha) d \theta=\frac{1}{2} A B \sin (\alpha-\beta)$
(iii) $\frac{1}{2 \pi} \int_{0}^{2 \pi} A \sin (m \theta+\alpha) \cdot B \cos (n \theta+\beta) d \theta=0$
(iv) $\frac{1}{2 \pi} \int_{0}^{2 \pi} A \sin (m \theta+\alpha) \cdot B \cos (n \theta+\beta) d \theta=0$

Result (iii) and (iv) implies that power is transferred between same harmonics of voltages and currents.
Thus integration of equation (i) gives.

$$
P=\frac{1}{2} E_{1} I_{1} \cos \phi+\frac{1}{2} E_{3} I_{3} \cos \phi_{3}
$$

## Q. 26 to Q. 55 carry two marks each.

MCQ 1.26 Given that

$$
\boldsymbol{A}=\left[\begin{array}{rr}
-5 & -3 \\
2 & 0
\end{array}\right] \text { and } \boldsymbol{I}=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] \text {, the value of } A^{3} \text { is }
$$

(A) $15 \boldsymbol{A}+12 \boldsymbol{I}$
(B) $19 \boldsymbol{A}+30 \boldsymbol{I}$
(C) $17 \boldsymbol{A}+15 \boldsymbol{I}$
(D) $17 \boldsymbol{A}+21 \boldsymbol{I}$

SOL 1.26 Option (B) is correct.
Characteristic equation.

Since characteristic equation satisfies its own matrix, so

$$
\boldsymbol{A}^{2}+5 \boldsymbol{A}+6=0 \Rightarrow \boldsymbol{A}^{2}=-5 \boldsymbol{A}-6 \boldsymbol{I}
$$

Multiplying with $\boldsymbol{A}$

$$
\begin{aligned}
\boldsymbol{A}^{3}+5 \boldsymbol{A}^{2}+6 \boldsymbol{A} & =0 \\
\boldsymbol{A}^{3}+5(-5 \boldsymbol{A}-6 \boldsymbol{I})+6 \boldsymbol{A} & =0 \\
\boldsymbol{A}^{3} & =19 \boldsymbol{A}+30 \boldsymbol{I}
\end{aligned}
$$

MCQ 1.27 The maximum value of $f(x)=x^{3}-9 x^{2}+24 x+5$ in the interval $[1,6]$ is
(A) 21
(B) 25
(C) 41
(D) 46

SOL 1.27 Option (B) is correct.

$$
\begin{aligned}
& f(x)=x^{3}-9 x^{2}+24 x+5 \\
& \frac{d f(x)}{d x}=3 x^{2}-18 x+24=0 \\
& \Rightarrow \quad \frac{d f(x)}{d x}=x^{2}-6 x+8=0 \\
& x=4, x=2
\end{aligned}
$$

$$
\frac{d^{2} f(x)}{d x^{2}}=6 x-18
$$

For $x=2, \frac{d^{2} f(x)}{d x^{2}}=12-18=-6<0$
So at $x=2, f(x)$ will be maximum

$$
\begin{aligned}
\left.f(x)\right|_{\max } & =(2)^{3}-9(2)^{2}+24(2)+5 \\
& =8-36+48+5 \\
& =25
\end{aligned}
$$

MCQ 1.28 If $V_{A}-V_{B}=6 \mathrm{~V}$ then $V_{C}-V_{D}$ is

$$
\begin{aligned}
& |\boldsymbol{A}-\lambda \boldsymbol{I}|=0 \\
& \left|\begin{array}{cc}
-5-\lambda & -3 \\
2 & -\lambda
\end{array}\right|=0 \\
& 5 \lambda+\lambda^{2}+6=0 \\
& \lambda^{2}+5 \lambda+6=0
\end{aligned}
$$


(A) -5 V
(B) 2 V
(C) 3 V
(D) 6 V

SOL 1.28 Option (A) is correct.
In the given circuit

$$
V_{A}-V_{B}=6 \mathrm{~V}
$$

So current in the branch will be

$$
I_{A B}=\frac{6}{2}=3 \mathrm{~A}
$$

We can see, that the circuit is a one port circuit looking from terminal $B D$ as shown below


For a one port network current entering one terminal, equals the current leaving the second terminal. Thus the outgoing current from $A$ to $B$ will be equal to the incoming current from $D$ to $C$ as shown
i.e. $\quad I_{D C}=I_{A B}=3 \mathrm{~A}$


The total current in the resistor $1 \Omega$ will be

| $I_{1}$ | $=2+I_{D C}$ | $\quad($ By writing KCL at node $D)$ |
| ---: | :--- | ---: |
|  | $=2+3=5 \mathrm{~A}$ |  |
| So, $\quad V_{C D}$ | $=1 \times\left(-I_{1}\right)$ |  |

$$
=-5 \mathrm{~V}
$$

MCQ 1.29 The voltage gain $A_{v}$ of the circuit shown below is

(A) $\left|A_{v}\right| \approx 200$
(B) $\left|A_{v}\right| \approx 100$
(C) $\left|A_{v}\right| \approx 20$
(D) $\left|A_{v}\right| \approx 10$

SOL 1.29 Option (D) is correct.

## DC Analysis :



Using KVL in input loop,

$$
\begin{align*}
V_{C}-100 I_{B}-0.7 & =0 \\
V_{C} & =100 I_{B}+0.7  \tag{i}\\
I_{C} & \simeq I_{E}=\frac{13.7-V_{C}}{12 k}=(\beta+1) I_{B} \\
\frac{13.7-V_{C}}{12 \times 10^{3}} & =100 I_{B} \tag{ii}
\end{align*}
$$

Solving equation (i) and (ii),

$$
I_{B}=0.01 \mathrm{~mA}
$$

## Small Signal Analysis :

Transforming given input voltage source into equivalent current source.


This is a shunt-shunt feedback amplifier.
Given parameters,

$$
\begin{aligned}
& r_{\pi}=\frac{V_{T}}{I_{B}}=\frac{25 \mathrm{mV}}{0.01 \mathrm{~mA}}=2.5 \mathrm{k} \Omega \\
& g_{m}=\frac{\beta}{r_{\pi}}=\frac{100}{2.5 \times 1000}=0.04 \mathrm{~s}
\end{aligned}
$$

Writing KCL at output node

$$
\begin{array}{r}
\frac{v_{0}}{R_{C}}+g_{m} v_{\pi}+\frac{v_{0}-v_{\pi}}{R_{F}}=0 \\
v_{0}\left[\frac{1}{R_{C}}+\frac{1}{R_{F}}\right]+v_{\pi}\left[g_{m}-\frac{1}{R_{F}}\right]=0
\end{array}
$$

Substituting $R_{C}=12 \mathrm{k} \Omega, R_{F}=100 \mathrm{k} \Omega, g_{m}=0.04 \mathrm{~s}$

$$
\begin{align*}
v_{0}\left(9.33 \times 10^{-5}\right)+v_{\pi}(0.04) & =0  \tag{i}\\
v_{0} & =-428.72 V_{\pi}
\end{align*}
$$

Writing KCL at input node

$$
\begin{aligned}
& \frac{v_{i}}{R_{s}}=\frac{v_{\pi}}{R_{s}}+\frac{v_{\pi}}{r_{\pi}}+\frac{v_{\pi}-v_{o}}{R_{F}} \\
& \frac{v_{i}}{R_{s}}=v_{\pi}\left[\frac{1}{R_{s}}+\frac{1}{r_{\pi}}+\frac{1}{R_{F}}\right]-\frac{v_{0}}{R_{F}} \\
& \frac{v_{i}}{R_{s}}=v_{\pi}\left(5.1 \times 10^{-4}\right)-\frac{v_{0}}{R_{F}}
\end{aligned}
$$

Substituting $V_{\pi}$ from equation (i)

$$
\begin{aligned}
\frac{v_{i}}{R_{s}} & =\frac{-5.1 \times 10^{-4}}{428.72} v_{0}-\frac{v_{0}}{R_{F}} \\
\frac{v_{i}}{10 \times 10^{3}} & =-1.16 \times 10^{-6} v_{0}-1 \times 10^{-5} v_{0} \quad R_{s}=10 \mathrm{k} \Omega \text { (source resistance) } \\
\frac{v_{i}}{10 \times 10^{3}} & =-1.116 \times 10^{-5} \\
\left|A_{v}\right| & =\left|\frac{v_{0}}{v_{i}}\right|=\frac{1}{10 \times 10^{3} \times 1.116 \times 10^{-5}} \simeq 8.96
\end{aligned}
$$

MCQ 1.30 The state transition diagram for the logic circuit shown is

(A)

(B)

(C)

(B)


SOL 1.30 Option (D) is correct.
Let $Q_{n+1}$ is next state and $Q_{n}$ is the present state. From the given below figure.

$$
\begin{aligned}
D & =\bar{Y}=\bar{A} X_{0}+A X_{1} \\
Q_{n+1} & =D=\bar{A} X_{0}+A X_{1} \\
Q_{n+1} & =\bar{A} \overline{Q_{n}}+A Q_{n} \\
Q_{n+1} & =\overline{Q_{n}} \\
Q_{n+1} & =Q_{n}
\end{aligned}
$$

$$
X_{0}=\bar{Q}, X_{1}=Q
$$

$$
\text { If } A=0,
$$

(toggle of previous state)
If $A=1$,
So state diagram is


MCQ 1.31 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n]=(1 / 2)^{n} u[n]$ and $g[n]$ is a causal sequence. If $y[0]=1$ and $y[1]=1 / 2$, then $g[1]$ equals
(A) 0
(B) $1 / 2$
(C) 1
(D) $3 / 2$

SOL 1.31 Option (A) id correct.
Convolution sum is defined as

$$
y[n]=h[n] * g[n]=\sum_{k=-\infty}^{\infty} h[n] g[n-k]
$$

For causal sequence, $\quad y[n]=\sum_{k=0}^{\infty} h[n] g[n-k]$

$$
y[n]=h[n] g[n]+h[n] g[n-1]+h[n] g[n-2]+\ldots \ldots
$$

For $n=0$,

$$
\begin{align*}
y[0] & =h[0] g[0]+h[1] g[-1]+\ldots \ldots \ldots . . \\
y[0] & =h[0] g[0] \\
y[0] & =h[0] g[0]
\end{align*} \quad g[-1]=g[-2]=\ldots 00
$$

For $n=1$,

$$
\begin{aligned}
y[1] & =h[1] g[1]+h[1] g[0]+h[1] g[-1]+\ldots & \\
y[1] & =h[1] g[1]+h[1] g[0] & \\
\frac{1}{2} & =\frac{1}{2} g[1]+\frac{1}{2} g[0] & \\
1 & =g[1]+g[0] & \left(\frac{1}{2}\right)^{1}=\frac{1}{2} \\
g[1] & =1-g[0] &
\end{aligned}
$$

From equation (i),

$$
g[0]=\frac{y[0]}{h[0]}=\frac{1}{1}=1
$$

So,

$$
g[1]=1-1=0
$$

MCQ 1.32 The circuit shown is a

(A) low pass filter with $f_{3 d B}=\frac{1}{\left(R_{1}+R_{2}\right) C} \mathrm{rad} / \mathrm{s}$
(B) high pass filter with $f_{3 d B}=\frac{1}{R_{1} C} \mathrm{rad} / \mathrm{s}$
(C) low pass filter with $f_{3 d B}=\frac{1}{R_{1} C} \mathrm{rad} / \mathrm{s}$
(D) high pass filter with $f_{3 d B}=\frac{1}{\left(R_{1}+R_{2}\right) C} \mathrm{rad} / \mathrm{s}$

SOL 1.32 Option (B) is correct.
First we obtain the transfer function.

$\frac{0-V_{i}(j \omega)}{\frac{1}{j \omega C}+R_{1}}+\frac{0-V_{o}(j \omega)}{R_{2}}=0$

$$
\frac{V_{o}(j \omega)}{R_{2}}=\frac{-V_{i}(j \omega)}{\frac{1}{j \omega C}+R_{1}}
$$

$$
V_{o}(j \omega)=-\frac{V_{i}(j \omega) R_{2}}{R_{1}-j \frac{1}{\omega C}}
$$

At $\omega \rightarrow 0$ (Low frequencies)

$$
\frac{1}{\omega C} \rightarrow \infty, \text { so } V_{o}=0
$$

At $\omega \rightarrow \infty$ (higher frequencies)

$$
\frac{1}{\omega C} \rightarrow 0, \text { so } V_{o}(j \omega)=-\frac{R_{2}}{R_{1}} V_{i}(j \omega)
$$

The filter passes high frequencies so it is a high pass filter.

$$
\begin{aligned}
H(j \omega) & =\frac{V_{o}}{V_{i}}=\frac{-R_{2}}{R_{1}-j \frac{1}{\omega C}} \\
|H(\infty)| & =\left|\frac{-R_{2}}{R_{1}}\right|=\frac{R_{2}}{R_{1}}
\end{aligned}
$$

At 3 dB frequency, gain will be $\sqrt{2}$ times of maximum gain $[H(\infty)]$

$$
\begin{aligned}
\left|H\left(j \omega_{0}\right)\right| & =\frac{1}{\sqrt{2}}|H(\infty)| \\
\frac{R_{2}}{\sqrt{R_{1}^{2}+\frac{1}{\omega_{0}^{2} C^{2}}}} & =\frac{1}{\sqrt{2}}\left(\frac{R_{2}}{R_{1}}\right) \\
2 R_{1}^{2} & =R_{1}^{2}+\frac{1}{\omega_{0}^{2} C^{2}} \\
R_{1}^{2} & =\frac{1}{\omega^{2} C^{2}} \\
\omega_{0} & =\frac{1}{R_{1} C}
\end{aligned}
$$

So,

MCQ 1.33 For the system below, $\boldsymbol{S}_{D 1}$ and $\boldsymbol{S}_{D 2}$ are complex power demands at bus 1 and bus 2 respectively. If $\left|\boldsymbol{V}_{2}\right|=1 \mathrm{pu}$, the VAR rating of the capacitor $\left(\boldsymbol{Q}_{G 2}\right)$ connected at
bus 2 is
Bus 1
Bus 2

(A) 0.2 pu
(B) 0.268 pu
(C) 0.312 pu
(D) 0.4 pu

SOL 1.33 Option (B) is correct.
Complex power for generator

$$
\boldsymbol{S}_{G_{1}}=\boldsymbol{S}_{D 1}+\boldsymbol{S}_{D 2}=1+1=2 \mathrm{pu}
$$

(Line is lossless)
Power transferred from bus 1 to bus 2 is 1 pu , so

$$
1=\frac{\left|\boldsymbol{V}_{1} \| \boldsymbol{V}_{2}\right| \sin \left(\theta_{1}-\theta_{2}\right)}{X}
$$



$$
\begin{aligned}
\left|\boldsymbol{V}_{1}\right| & =\left|\boldsymbol{V}_{2}\right|=1 \mathrm{pu} \\
X & =0.5 \mathrm{pu}
\end{aligned}
$$

$$
\left(\theta_{1}=0^{\circ}\right)
$$

So,

$$
\boldsymbol{V}_{1}=1 \angle 0^{\circ} \mathrm{V}
$$

$$
\boldsymbol{V}_{2}=1 /-30^{\circ} \mathrm{V}
$$

Current,

$$
\begin{aligned}
\boldsymbol{I}_{12} & =\frac{\boldsymbol{V}_{1}-\boldsymbol{V}_{2}}{Z}=\frac{1 / 0^{\circ}-1 / 30^{\circ}}{j 0.5} \\
& =(1-j 0.288) \mathrm{pu}
\end{aligned}
$$

Current in $\boldsymbol{S}_{D_{2}}$ is $\boldsymbol{I}_{2}$,

$$
\begin{aligned}
\boldsymbol{S}_{D_{2}} & =\boldsymbol{V}_{2} \boldsymbol{I}_{2}^{*} \\
1 & =1 \angle-30^{\circ} \boldsymbol{I}_{2}^{*} \\
\boldsymbol{I}_{2} & =1 \angle-30^{\circ} \mathrm{pu}
\end{aligned}
$$

Current in $\boldsymbol{Q}_{G_{2}}$,

$$
\begin{aligned}
\boldsymbol{I}_{G} & =\boldsymbol{I}_{2}-\boldsymbol{I}_{12} \\
& =1 \angle-30^{\circ}-(1-j 0.288) \\
& =0.268 \angle-120^{\circ}
\end{aligned}
$$

VAR rating of capacitor,

$$
Q_{C}=\left|\boldsymbol{V}_{2} \| \boldsymbol{V}_{G}\right|=1 \times 0.268=0.268 \mathrm{pu}
$$

MCQ 1.34 A cylinder rotor generator delivers 0.5 pu power in the steady-state to an infinite
bus through a transmission line of reactance 0.5 pu . The generator no-load voltage is 1.5 pu and the infinite bus voltage is 1 pu . The inertia constant of the generator is $5 \mathrm{MW}-\mathrm{s} / \mathrm{MVA}$ and the generator reactance is 1 pu . The critical clearing angle, in degrees, for a three-phase dead short circuit fault at the generator terminal is
(A) 53.5
(B) 60.2
(C) 70.8
(D) 79.6

SOL 1.34 Option (D) is correct.


Total reactance, $\quad X=j 1+j 0.5=j 1.5 \mathrm{pu}$
Critical angle is given as,

$$
\begin{equation*}
\delta_{c r}=\cos ^{-1}\left[\left(\pi-2 \delta_{0}\right) \sin \delta_{0}-\cos \delta_{0}\right] \tag{i}
\end{equation*}
$$

$\delta_{0} \rightarrow$ steady state torque angle.
Steady state power is given as
where,


$$
0.5=\frac{(1.5)(1)}{1.5} \sin \delta_{0} \quad P_{m}=0.5 \mathrm{pu}
$$

$$
\sin \delta_{0}=0.5
$$

$$
\delta_{0}=30^{\circ}
$$

In radian,

$$
\delta_{0}=30^{\circ} \times \frac{\pi}{180^{\circ}}=0.523
$$

Substituting $\delta_{0}$ into equation (i)

$$
\begin{aligned}
\delta_{c r} & =\cos ^{-1}\left[(\pi-2 \times 0.523) \sin 30^{\circ}-\cos 30^{\circ}\right] \\
& =\cos ^{-1}[(2.095)(0.5)-0.866] \\
& =\cos ^{-1}(0.1815) \\
& \simeq 79.6^{\circ}
\end{aligned}
$$

MCQ 1.35 In the circuit shown, an ideal switch $S$ is operated at 100 kHz with a duty ratio of $50 \%$. Given that $\Delta i_{c}$ is 1.6 A peak-to-peak and $I_{0}$ is 5 A dc, the peak current in $S$ , is

(A) 6.6 A
(B) 5.0 A
(C) 5.8 A
(D) 4.2 A

SOL 1.35 Option (C) is correct.

$$
I_{S}=I_{0}+\frac{\Delta i_{c}}{2}=5+0.8=5.8 \mathrm{~A}
$$

MCQ 1.36 A $220 \mathrm{~V}, 15 \mathrm{~kW}, 100 \mathrm{rpm}$ shunt motor with armature resistance of $0.25 \Omega$, has a rated line current of 68 A and a rated field current of 2.2 A . The change in field flux required to obtain a speed of 1600 rpm while drawing a line current of 52.8 A and a field current of 1.8 A is
(A) $18.18 \%$ increase
(C) $36.36 \%$ increase


SOL 1.36 Option (D) is correct.
$E \propto n \phi$
where $n \rightarrow$ speed, $\phi \rightarrow$ flux and $E \rightarrow$ back emf Given that,

$$
\begin{aligned}
V_{t} & =250 \mathrm{~V}, R_{a}=0.25 \Omega \\
n_{1} & =1000 \mathrm{rpm}, I_{L 1}=68 \mathrm{~A}, I_{F 1}=2.2 \mathrm{~A} \\
I_{a 1} & =I_{L 1}-I_{F 1}=68-2.2=65.8 \mathrm{~A} \\
E_{1} & =V_{t}-I_{a}, R_{a} \\
& =250-(65.8)(0.25) \\
& =203.55 \mathrm{~V}
\end{aligned}
$$

Armature current,

Now,
Armature current,

$$
\begin{aligned}
n_{2} & =1600 \mathrm{rpm}, I_{L 2}=52.8 \mathrm{~A}, I_{F 2}=1.8 \mathrm{~A} \\
I_{a_{2}} & =I_{L 2}-I_{F 2} \\
& =52.8-1.8 \\
& =51 \mathrm{~A} \\
E_{2} & =V_{t}-I_{a_{2}} R_{a} \\
& =220-(51)(0.25) \\
& =207.25 \mathrm{~V} \\
\frac{E_{1}}{E_{2}} & =\left(\frac{n_{1}}{n_{2}}\right)\left(\frac{\phi_{1}}{\phi_{2}}\right) \\
\frac{203.55}{207.45} & =\left(\frac{1000}{1600}\right)\left(\frac{\phi_{1}}{\phi_{2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\phi_{2} & =0.6369 \phi_{1} \\
\% \text { reduce in flux } & =\frac{\phi_{1}-\phi_{2}}{\phi_{1}} \times 100 \\
& =\frac{\phi_{1}-0.6369 \phi_{1}}{\phi_{1}} \times 100 \\
& \simeq 36.3 \%
\end{aligned}
$$

MCQ 1.37 A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is
(A) $1 / 3$
(B) $1 / 2$
(C) $2 / 3$
(D) $3 / 4$

SOL 1.37 Option (C) is correct.
Probability of appearing a head is $1 / 2$. If the number of required tosses is odd, we have following sequence of events.

Probability

$$
\begin{aligned}
& H, \text { TTH, TTTTH, } \ldots \ldots \ldots . . \\
& P=\frac{1}{2}+\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{5}+\ldots . .
\end{aligned}
$$



MCQ 1.38 The direction of vector $\boldsymbol{A}$ is radially outward from the origin, with $|\boldsymbol{A}|=k r^{n}$. where $r^{2}=x^{2}+y^{2}+z^{2}$ and $k$ is a constant. The value of $n$ for which $\nabla \cdot \boldsymbol{A}=0$ is
(A) -2
(B) 2
(C) 1
(D) 0

SOL 1.38 Option (A) is correct.
Divergence of $\boldsymbol{A}$ in spherical coordinates is given as

$$
\begin{aligned}
\nabla \cdot \boldsymbol{A} & =\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} A_{r}\right) \\
& =\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(k r^{n+2}\right) \\
& =\frac{k}{r^{2}}(n+2) r^{n+1} \\
& =k(n+2) r^{n-1}=0 \quad \text { (given) } \\
n+2 & =0 \\
n & =-2
\end{aligned}
$$

MCQ 1.39 Consider the differential equation

$$
\frac{d^{2} y(t)}{d t^{2}}+2 \frac{d y(t)}{d t}+y(t)=\delta(t) \text { with }\left.y(t)\right|_{t=0^{-}}=-2 \text { and }\left.\frac{d y}{d t}\right|_{t=0^{-}}=0
$$

The numerical value of $\left.\frac{d y}{d t}\right|_{t=0^{+}}$is
(A) -2
(B) -1
(C) 0
(D) 1

SOL 1.39 Option (D) is correct.

$$
\frac{d^{2} y(t)}{d t^{2}}+\frac{2 d y(t)}{d t}+y(t)=\delta(t)
$$

By taking Laplace transform with initial conditions

$$
\begin{gathered}
{\left[s^{2} Y(s)-s y(0)-\left.\frac{d y}{d t}\right|_{t=0}\right]+2[s y(s)-y(0)]+Y(s)=1} \\
\Rightarrow \quad\left[s^{2} Y(s)+2 s-0\right]+2[s Y(s)+2]+Y(s)=1 \\
Y(s)\left[s^{2}+2 s+1\right]=1-2 s-4 \\
Y(s)=\frac{-2 s-3}{s^{2}+2 s+1}
\end{gathered}
$$

We know that,
If,

then,

So,

$$
\begin{aligned}
\frac{d y(t)}{d t} & \stackrel{\mathcal{L}}{\leftrightarrows} s Y(s)-y(0) \\
s Y(s)-y(0) & =\frac{(-2 s-3) s}{\left(s^{2}+2 s+1\right)}+2 \\
& =\frac{-2 s^{2}-3 s+2 s^{2}+4 s+2}{\left(s^{2}+2 s+1\right)} \\
s Y(s)-y(0) & =\frac{s+2}{(s+1)^{2}}=\frac{s+1}{(s+1)^{2}}+\frac{1}{(s+1)^{2}} \\
& =\frac{1}{s+1}+\frac{1}{(s+1)^{2}}
\end{aligned}
$$

By taking inverse Laplace transform

$$
\frac{d y(t)}{d t}=e^{-t} u(t)+t e^{-t} u(t)
$$

At $t=0^{+}$,

$$
\left.\frac{d y}{d t}\right|_{t=0^{+}}=e^{0}+0=1
$$

MCQ 1.40
Assuming both the voltage sources are in phase, the value of $R$ for which maximum power is transferred from circuit $A$ to circuit $B$ is


Circuit $A: \begin{gathered}\text { Circuit } B\end{gathered}$
(A) $0.8 \Omega$
(B) $1.4 \Omega$
(C) $2 \Omega$
(D) $2.8 \Omega$

SOL 1.40 Option (A) is correct.
We obtain Thevenin equivalent of circuit $B$.


Thevenin Impedance :


Current in the circuit, $\quad I_{1}=\frac{10-3}{2+R}$
Power transfer from circuit $A$ to $B$

$$
\begin{aligned}
& P=\left(I_{1}^{2}\right)^{2} R+3 I_{1} \\
& P=\left[\frac{10-3}{2+R}\right]^{2} R+3\left[\frac{10-3}{2+R}\right] \\
& P=\frac{49 R}{(2+R)^{2}}+\frac{21}{(2+R)} \\
& P=\frac{49 R+21(2+R)}{(2+R)^{2}}
\end{aligned}
$$

$$
\begin{gathered}
P=\frac{42+70 R}{(2+R)^{2}} \\
\frac{d P}{d R}=\frac{(2+R)^{2} 70-(42+70 R) 2(2+R)}{(2+R)^{4}}=0 \\
(2+R)[(2+R) 70-(42+70 R) 2]=0 \\
140+70 R-84-140 R=0 \\
56=70 R \\
R=0.8 \Omega
\end{gathered}
$$

MCQ 1.41 The state variable description of an LTI system is given by

$$
\begin{aligned}
\left(\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2} \\
\dot{x}_{3}
\end{array}\right) & =\left(\begin{array}{rrr}
0 & a_{1} & 0 \\
0 & 0 & a_{2} \\
a_{3} & 0 & 0
\end{array}\right)\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right)+\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right) u \\
y & =\left(\begin{array}{lll}
1 & 0 & 0
\end{array}\right)\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right)
\end{aligned}
$$

where $y$ is the output and $u$ is the input. The system is controllable for
(A) $a_{1} \neq 0, a_{2}=0, a_{3} \neq 0$
(B) $a_{1}=0, a_{2} \neq 0, a_{3} \neq 0$
(C) $a_{1}=0, a_{3} \neq 0, a_{3}=0$ $\square$ (D) $a_{1} \neq 0, a_{2} \neq 0, a_{3}=0$

SOL 1.41 Option (D) is correct.
General form of state equations are given as

$$
\begin{aligned}
& \dot{x}=\boldsymbol{A} x+\boldsymbol{B} u \\
& \dot{y}=\boldsymbol{C} x+\boldsymbol{D} u
\end{aligned}
$$

For the given problem

$$
\begin{aligned}
\boldsymbol{A} & =\left[\begin{array}{rrr}
0 & a_{1} & 0 \\
0 & 0 & a_{2} \\
a_{3} & 0 & 0
\end{array}\right], \quad \boldsymbol{B}=\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right] \\
\boldsymbol{A} \boldsymbol{B} & =\left[\begin{array}{rrr}
0 & a_{1} & 0 \\
0 & 0 & a_{2} \\
a_{3} & 0 & 0
\end{array}\right]\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\left[\begin{array}{c}
0 \\
a_{2} \\
0
\end{array}\right] \\
\boldsymbol{A}^{2} \boldsymbol{B} & =\left[\begin{array}{rrr}
0 & 0 & a_{1} a_{2} \\
a_{2} a_{3} & 0 & 0 \\
0 & a_{3} a_{1} & 0
\end{array}\right]\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]=\left[\begin{array}{r}
a_{1} a_{2} \\
0 \\
0
\end{array}\right]
\end{aligned}
$$

For controllability it is necessary that following matrix has a tank of $n=3$.

So,

$$
\begin{aligned}
\boldsymbol{U} & =\left[\boldsymbol{B}: \boldsymbol{A} \boldsymbol{B}: \boldsymbol{A}^{2} \boldsymbol{B}\right] \\
& =\left[\begin{array}{rrr}
0 & 0 & a_{1} a_{2} \\
0 & a_{2} & 0 \\
1 & 0 & 0
\end{array}\right] \\
a_{2} & \neq 0
\end{aligned}
$$

$$
a_{1} a_{2} \neq 0 \Rightarrow a_{1} \neq 0
$$

$a_{3}$ may be zero or not.
MCQ 1.42 The Fourier transform of a signal $h(t)$ is $H(j \omega)=(2 \cos \omega)(\sin 2 \omega) / \omega$. The value of $h(0)$ is
(A) $1 / 4$
(B) $1 / 2$
(C) 1
(D) 2
sOL 1.42 Option (C) is correct.

$$
\begin{aligned}
H(j \omega) & =\frac{(2 \cos \omega)(\sin 2 \omega)}{\omega} \\
& =\frac{\sin 3 \omega}{\omega}+\frac{\sin \omega}{\omega}
\end{aligned}
$$

We know that inverse Fourier transform of $\sin c$ function is a rectangular function.


So, inverse Fourier transform of $H(j \omega)$

$$
\begin{aligned}
h(t) & =h_{1}(t)+h_{2}(t) \\
h(0) & =h_{1}(0)+h_{2}(0) \\
& =\frac{1}{2}+\frac{1}{2}=1
\end{aligned}
$$

MCQ 1.43 The feedback system shown below oscillates at $2 \mathrm{rad} / \mathrm{s}$ when

(A) $K=2$ and $a=0.75$
(B) $K=3$ and $a=0.75$
(C) $K=4$ and $a=0.5$
(D) $K=2$ and $a=0.5$
sOL 1.43 Option (A) is correct.

$$
\begin{aligned}
& Y(s)=\frac{K(s+1)}{s^{3}+a s^{2}+2 s+1}[R(s)-Y(s)] \\
& Y(s)\left[1+\frac{K(s+1)}{s^{3}+a s^{2}+2 s+1}\right]=\frac{K(s+1)}{s^{3}+a s^{2}+2 s+1} R(s) \\
& Y(s)\left[s^{3}+a s^{2}+s(2+k)+(1+k)\right]=K(s+1) R(s)
\end{aligned}
$$

Transfer Function

$$
H(s)=\frac{Y(s)}{R(s)}=\frac{K(s+1)}{s^{3}+a s^{2}+s(2+k)+(1+k)}
$$

## Routh Table :

| $s^{3}$ | 1 | $2+K$ |
| :---: | :---: | :---: |
| $s^{2}$ | $a$ | $1+K$ |
| $s^{1}$ | $\frac{a(2+K)-(1+K)}{a}$ | 0 |

For oscillation,

$$
\frac{a(2+K)-(1+K)}{a}=0
$$



$$
\begin{aligned}
& \qquad a=\frac{K+1}{K+2} \square \square \\
& \text { Auxiliary equation }
\end{aligned}
$$

$$
\begin{aligned}
A(s) & =a s^{2}+(k+1)=0 \\
s^{2} & =-\frac{k+1}{a}
\end{aligned}
$$

$$
s^{2}=\frac{-k+1}{(k+1)}(k+2)
$$

$$
s^{2}=-(k+2)
$$

$$
s=j \sqrt{k+2}
$$

$$
j \omega=j \sqrt{k+2}
$$

$$
\omega=\sqrt{k+2}=2
$$

(Oscillation frequency)

$$
k=2
$$

and

$$
a=\frac{2+1}{2+2}=\frac{3}{4}=0.75
$$

MCQ 1.44 The input $x(t)$ and output $y(t)$ of a system are related as $y(t)=\int_{-\infty}^{t} x(\tau) \cos (3 \tau) d \tau$ . The system is
(A) time-invariant and stable
(B) stable and not time-invariant
(C) time-invariant and not stable
(D) not time-invariant and not stable

SOL 1.44 Option (D) is correct.

$$
y(t)=\int_{-\infty}^{t} x(\tau) \cos (3 \tau) d \tau
$$

Time invariance :

Let,

$$
\begin{aligned}
x(t) & =\delta(t) \\
y(t) & =\int_{-\infty}^{t} \delta(t) \cos (3 \tau) d \tau \\
& =u(t) \cos (0) \\
& =u(t)
\end{aligned}
$$

For a delayed input $\left(t-t_{0}\right)$ output is

$$
\begin{aligned}
y\left(t, t_{0}\right) & =\int_{-\infty}^{t} \delta\left(t-t_{0}\right) \cos (3 \tau) d \tau \\
& =u(t) \cos \left(3 t_{0}\right)
\end{aligned}
$$

Delayed output

$$
\begin{aligned}
y\left(t-t_{0}\right) & =u\left(t-t_{0}\right) \\
y\left(t, t_{0}\right) & \neq y\left(t-t_{0}\right)
\end{aligned}
$$

System is not time invariant.

## Stability :

Consider a bounded input $x(t)=\cos 3 t$

$$
\begin{aligned}
y(t) & =\int_{-\infty}^{t} \cos ^{2} 3 t=\int_{-\infty}^{t} \frac{1-\cos 6 t}{2} \\
& =\frac{1}{2} \int_{-\infty}^{t} 1 d t-\frac{1}{2} \int_{-\infty}^{t} \cos 6 t d t
\end{aligned}
$$

As $t \rightarrow \infty, y(t) \rightarrow \infty$ (unbounded)
System is not stable.
MCQ 1.45 An analog voltmeter uses external multiplier settings. With a multiplier setting of $20 \mathrm{k} \Omega$, it reads 440 V and with a multiplier setting of $80 \mathrm{k} \Omega$, it reads 352 V . For a multiplier setting of $40 \mathrm{k} \Omega$, the voltmeter reads
(A) 371 V
(B) 383 V
(C) 394 V
(D) 406 V

SOL 1.45 Option (D) is correct.
A voltmeter with a multiplier is shown in figure below.


Here $\quad I_{m}=$ Fully scale deflection current of meter.
$R_{m}=$ Internal resistance of meter
$R_{s}=$ Voltage across the meter

$$
\begin{align*}
V & =\text { Full range voltage of instrument } \\
V_{m} & =I_{m} R_{m} \\
V & =I_{m}\left(R_{m}+R_{s}\right) \\
\frac{V}{V_{m}} & =\frac{R_{m}+R_{s}}{R_{m}}=1+\frac{R_{s}}{R_{m}} \\
R_{s 1} & =20 \mathrm{k} \Omega, V_{m 1}=440 \mathrm{~V} \tag{i}
\end{align*}
$$

Here when,
So, $\quad \frac{V}{440}=1+\frac{20 \mathrm{k}}{R_{m}}$
When, $\quad R_{s 2}=80 \mathrm{k} \Omega, V_{m 2}=352 \mathrm{~V}$
So, $\quad \frac{V}{352}=1+\frac{80 \mathrm{k}}{R_{m}}$
Solving equation (i) and (ii), we get

$$
V=480 \mathrm{~V}, R_{m}=220 \mathrm{k} \Omega
$$

So when

$$
\begin{aligned}
R_{s 3} & =40 \mathrm{k} \Omega, V_{m 3}=? \\
\frac{480}{V_{m 3}} & =1+\frac{40 \mathrm{k}}{220 \mathrm{k}} \\
V_{m 2} & \simeq 406 \mathrm{~V}
\end{aligned}
$$

MCQ 1.46 The locked rotor current in a 3-phase, star connected $15 \mathrm{~kW}, 4$ pole, $230 \mathrm{~V}, 50 \mathrm{~Hz}$ induction motor at rated conditions is 50 A . Neglecting losses and magnetizing current, the approximate locked rotorline current drawn when the motor is connected to a $236 \mathrm{~V}, 57 \mathrm{~Hz}$ supply is
(A) 58.5 A
(B) 45.0 A
(C) 42.7 A
(D) 55.6 A

SOL 1.46 Option (B) is correct.
Given that magnetizing current and losses are to be neglected. Locked rotor line current.

$$
\begin{align*}
I_{2} & =\frac{E_{2}}{Z_{2}}=\frac{E_{2}}{\sqrt{R_{2}^{2}+X_{2}^{2}}}  \tag{2}\\
I_{2} & =\frac{E_{2}}{X_{2}} \\
I_{2} & =\frac{E_{2}}{\omega L_{2}} \\
\text { So } \quad I_{2} & \propto \frac{E_{2}}{f} \\
& \frac{50}{I_{2}^{\prime}} \\
& =\left(\frac{230}{236}\right)\left(\frac{57}{50}\right) \\
I_{2}^{\prime} & =45.0 \mathrm{~A}
\end{align*}
$$

MCQ 1.47 A single phase $10 \mathrm{kVA}, 50 \mathrm{~Hz}$ transformer with 1 kV primary winding draws 0.5 A
and 55 W , at rated voltage and frequency, on no load. A second transformer has a core with all its linear dimensions $\sqrt{2}$ times the corresponding dimensions of the first transformer. The core material and lamination thickness are the same in both transformer. The primary winding of both the transformers have the save number of turns. If a rate voltage of 2 kV at 50 Hz is applied to the primary of the second transformer, then the no load current and power, respectively, are
(A) $0.7 \mathrm{~A}, 77.8 \mathrm{~A}$
(B) $0.7 \mathrm{~A}, 155.6 \mathrm{~W}$
(C) $1 \mathrm{~A}, 110 \mathrm{~W}$
(D) $1 \mathrm{~A}, 220 \mathrm{~W}$

SOL 1.47 Option (B) is correct.
Since the core length of the second transformer is $\sqrt{2}$ times of the first, so the core area of the second transformer is twice of the first.
Let subscript 1 is used for first transformer and 2 is used for second transform.
Area

$$
\begin{aligned}
& a_{2}=2 a_{1} \\
& l_{2}=\sqrt{2} l_{1} \\
& L=\frac{N^{2} \mu a}{l} \\
& N=\text { no. of turns } \\
& \mu=\text { length of flux path } \\
& a=\text { cross section area } \\
& l=\text { length } \\
& L \propto \frac{a}{l} \\
& \frac{L_{1}}{L_{2}}=\frac{a_{1}}{a_{2}} \cdot \frac{l_{2}}{l_{1}} \\
& \frac{L_{1}}{L_{2}}=\frac{a_{1}}{2 a_{1}} \cdot \frac{\sqrt{2} l_{1}}{l_{1}} \\
& L_{2}=\sqrt{2} L_{1}
\end{aligned}
$$

Length
Magnetizing inductance,
$N$ and $\mu$ are same for both the transformer

Thus, magnetizing reactance of second transformer is $\sqrt{2}$ times of first.
Magnetizing current $\quad X_{m 2}=\sqrt{2} X_{m 1}$

$$
I_{m}=\frac{V}{X_{m}}
$$

$$
\frac{I_{m 1}}{I_{m 2}}=\frac{V_{1}}{V_{2}} \cdot \frac{X_{m 2}}{X_{m 1}}
$$

$$
=\left(\frac{V_{1}}{2 V_{1}}\right)\left(\frac{\sqrt{2} X_{m 1}}{X_{m 1}}\right)
$$

$$
\left(V_{2}=2 V_{1}\right)
$$

$$
I_{m 2}=\sqrt{2} I_{m 1}
$$

Thus, magnetizing current of second transformer

$$
I_{m 2}=\sqrt{2} \times 0.5=0.707 \mathrm{~A}
$$

Since voltage of second transformer is twice that of first and current is $\sqrt{2}$ times that of first, so power will be $2 \sqrt{2}$ times of first transformer.

$$
P_{2}=2 \sqrt{2} \times 55=155.6 \mathrm{~W}
$$

## Common Data Question

## Common Data for Questions 48 and 49

In the 3 -phase inverter circuit shown, the load is balanced and the gating scheme is $180^{\circ}$ conduction mode. All the switching devices are ideal.


MCQ 1.48 The rms value of load phase voltage is
(A) 106.1 V
(B) 141.4 V
(C) 212.2 V
(D) 282.8 V

SOL 1.48 Option (B) is correct.
For a three-phase bridge inverter, rms value of output line voltage is

$$
\begin{aligned}
V_{L} & =\frac{\sqrt{2}}{3} V_{d c} \square \\
& =\frac{\sqrt{2}}{3} \times 300 \\
& =141.4 \mathrm{~V}
\end{aligned}
$$

MCQ 1.49 If the dc bus voltage $V_{d}=300 \mathrm{~V}$, the power consumed by 3-phase load is
(A) 1.5 kW
(B) 2.0 kW
(C) 2.5 kW
(D) 3.0 kW

SOL 1.49 Option (D) is correct.

$$
\begin{aligned}
P & =3 \times \frac{V_{L}^{2}}{R} \\
& =3 \times \frac{(141.4)^{2}}{20} \\
& \simeq 3 \mathrm{~kW}
\end{aligned}
$$

## Common Data for Questions 50 and 51 :

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed :
(i) $1 \Omega$ connected at port $B$ draws a current of 3 A
(ii) $2.5 \Omega$ connected at port $B$ draws a current of 2 A


MCQ 1.50 With 10 V dc connected at port $A$, the current drawn by $7 \Omega$ connected at port $B$ is
(A) $3 / 7 \mathrm{~A}$
(B) $5 / 7 \mathrm{~A}$
(C) 1 A
(D) $9 / 7 \mathrm{~A}$

SOL 1.50 Option (C) is correct.
When 10 V is connected at port $A$ the network is


Now, we obtain Thevenin equivalent for the circuit seen at load terminal, let Thevenin voltage is $V_{T h, 10 \mathrm{v}}$ with 10 V applied at port $A$ and Thevenin resistance is $R_{T h}$.


$$
I_{L}=\frac{V_{T h, 10 \mathrm{v}}}{R_{T h}+R_{L}}
$$

For $R_{L}=1 \Omega, I_{L}=3 \mathrm{~A}$

$$
\begin{equation*}
3=\frac{V_{T h, 10 \mathrm{~V}}}{R_{T h}+1} \tag{i}
\end{equation*}
$$

For $R_{L}=2.5 \Omega, I_{L}=2 \mathrm{~A}$

$$
\begin{equation*}
=\frac{V_{T h, 10 \mathrm{v}}}{R_{T h}+2.5} \tag{ii}
\end{equation*}
$$

Dividing above two

$$
\begin{aligned}
\frac{3}{2} & =\frac{R_{T h}+2.5}{R_{T h}+1} \\
3 R_{T h}+3 & =2 R_{T h}+5
\end{aligned}
$$

$$
R_{T h}=2 \Omega
$$

Substituting $R_{T h}$ into equation (i)

$$
V_{T h, 10 \mathrm{~V}}=3(2+1)=9 \mathrm{~V}
$$

Note that it is a non reciprocal two port network. Thevenin voltage seen at port $B$ depends on the voltage connected at port $A$. Therefore we took subscript $V_{T h, 10 \mathrm{v}}$. This is Thevenin voltage only when 10 V source is connected at input port $A$. If the voltage connected to port $A$ is different, then Thevenin voltage will be different. However, Thevenin's resistance remains same.
Now, the circuit is


For $R_{L}=7 \Omega$

$$
I_{L}=\frac{V_{T h, 10 \mathrm{~V}}}{2+R_{L}}=\frac{9}{2+7}=1 \mathrm{~A}
$$

MCQ 1.51 For the same network, with $6 \vee$ dc connected at port $A, 1 \Omega$ connected at port $B$ draws $7 / 3 \mathrm{~A}$. If 8 V dc is connected to port $A$, the open circuit voltage at port $B$ is $\begin{array}{ll}\text { (A) } 6 \mathrm{~V} & \text { (B) } 7 \mathrm{~V} \\ \text { (C) } 8 \mathrm{~V} & (\mathrm{D}) 9 \mathrm{~V}\end{array}$

SOL 1.51 Option (B) is correct.
Now, when 6 V connected at port $A$ let Thevenin voltage seen at port $B$ is $V_{T h, 6 \mathrm{~V}}$. Here $R_{L}=1 \Omega$ and $I_{L}=\frac{7}{3} \mathrm{~A}$


$$
\begin{aligned}
V_{T h, 6 \mathrm{~V}} & =R_{T h} \times \frac{7}{3}+1 \times \frac{7}{3} \\
& =2 \times \frac{7}{3}+\frac{7}{3}=7 \mathrm{~V}
\end{aligned}
$$

This is a linear network, so $V_{T h}$ at port $B$ can be written as

$$
V_{T h}=V_{1} \alpha+\beta
$$

where $V_{1}$ is the input applied at port $A$.
We have $V_{1}=10 \mathrm{~V}, V_{T h, 10 \mathrm{v}}=9 \mathrm{~V}$

$$
\begin{equation*}
\therefore \quad 9=10 \alpha+\beta \tag{i}
\end{equation*}
$$

When $V_{1}=6 \mathrm{~V}, V_{T h, 6 \mathrm{v}}=9 \mathrm{~V}$
$\therefore \quad 7=6 \alpha+\beta$
Solving (i) and (ii)

$$
\alpha=0.5, \beta=4
$$

Thus, with any voltage $V_{1}$ applied at port $A$, Thevenin voltage or open circuit voltage at port $B$ will be
So,

$$
\begin{aligned}
V_{T h, V_{1}} & =0.5 V_{1}+4 \\
V_{1} & =8 \mathrm{~V} \\
V_{T h, 8 \mathrm{~V}} & =0.5 \times 8+4=8=V_{o c}
\end{aligned}
$$

(open circuit voltage)

## Linked Answer Question

## Statement for Linked Answer Questions 52 and 53 :

In the circuit shown, the three voltmeter readings are $V_{1}=220 \mathrm{~V}, V_{2}=122 \mathrm{~V}$, $V_{3}=136 \mathrm{~V}$.


MCQ 1.52 The power factor of the load is
(A) 0.45
(B) 0.50
(C) 0.55
(D) 0.60

SOL 1.52 Option (A) is correct.
By taking $\boldsymbol{V}_{1}, \boldsymbol{V}_{2}$ and $\boldsymbol{V}_{3}$ all are phasor voltages.

$$
\boldsymbol{V}_{1}=\boldsymbol{V}_{2}+\boldsymbol{V}_{3}
$$

Magnitude of $\boldsymbol{V}_{1}, \boldsymbol{V}_{2}$ and $\boldsymbol{V}_{3}$ are given as

$$
V_{1}=220 \mathrm{~V}, V_{2}=122 \mathrm{~V}, V_{3}=136 \mathrm{~V}
$$

Since voltage across $R$ is in same phase with $\boldsymbol{V}_{1}$ and the voltage $V_{3}$ has a phase difference of $\theta$ with voltage $\boldsymbol{V}_{1}$, we write in polar form

$$
\begin{aligned}
\boldsymbol{V}_{1} & =V_{2} / 0^{\circ}+V_{3} \angle \theta \\
\boldsymbol{V}_{1} & =V_{2}+V_{3} \cos \theta+j V_{3} \sin \theta \\
\boldsymbol{V}_{1} & =\left(V_{2}+V_{3} \cos \theta\right)+j V_{3} \sin \theta \\
\left|\boldsymbol{V}_{1}\right| & =\sqrt{\left(V_{2}+V_{3} \cos \theta\right)^{2}+\left(V_{2} \sin \theta\right)^{2}} \\
220 & =\sqrt{(122+136 \cos \theta)^{2}+(136 \sin \theta)^{2}}
\end{aligned}
$$

By solving, power factor

$$
\cos \theta=0.45
$$

MCQ 1.53 If $R_{L}=5 \Omega$, the approximate power consumption in the load is
(A) 700 W
(B) 750 W
(C) 800 W
(D) 850 W

SOL 1.53 Option (B) is correct.
Voltage across load resistance

$$
\begin{aligned}
V_{R L} & =V_{3} \cos \theta \\
& =136 \times 0.45 \\
& =61.2 \mathrm{~V}
\end{aligned}
$$

Power absorbed in $R_{L}$

$$
P_{L}=\frac{V_{R L}^{2}}{R_{L}}=\frac{(61.2)^{2}}{5} \simeq 750 \mathrm{~W}
$$

## Statement for Linked Answer Questions 54 and 55 :

The transfer function of a compensator is given as

MCQ 1.54

$$
G_{c}(s)=\frac{s+a}{s+b}
$$

$G_{c}(s)$ is a lead compensator if
(A) $a=1, b=2$
(C) $a=-3, b=-1$


SOL 1.54 Option (A) is correct.

$$
G_{C}(s)=\frac{s+a}{s+b}=\frac{j \omega+a}{j \omega+b}
$$

Phase lead angle

$$
\begin{aligned}
\phi & =\tan ^{-1}\left(\frac{\omega}{a}\right)-\tan ^{-1}\left(\frac{\omega}{b}\right) \\
\phi & =\tan ^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^{2}}{a b}}\right) \\
& =\tan ^{-1}\left(\frac{\omega(b-a)}{a b+\omega^{2}}\right)
\end{aligned}
$$

For phase-lead compensation $\phi>0$

$$
\begin{aligned}
b-a & >0 \\
b & >a
\end{aligned}
$$

Note: For phase lead compensator zero is nearer to the origin as compared to pole, so option (C) can not be true.

MCQ 1.55 The phase of the above lead compensator is maximum at
(A) $\sqrt{2} \mathrm{rad} / \mathrm{s}$
(B) $\sqrt{3} \mathrm{rad} / \mathrm{s}$
(C) $\sqrt{6} \mathrm{rad} / \mathrm{s}$
(D) $1 / \sqrt{3} \mathrm{rad} / \mathrm{s}$

SOL 1.55 Option (A) is correct.

$$
\begin{aligned}
\phi & =\tan ^{-1}\left(\frac{\omega}{a}\right)-\tan ^{-1}\left(\frac{\omega}{b}\right) \\
\frac{d \phi}{d \omega} & =\frac{1 / a}{1+\left(\frac{\omega}{a}\right)^{2}}-\frac{1 / b}{1+\left(\frac{\omega}{b}\right)^{2}}=0 \\
\frac{1}{a}+\frac{\omega^{2}}{a b^{2}} & =\frac{1}{b}+\frac{1}{b} \frac{\omega^{2}}{a^{2}} \\
\frac{1}{a}-\frac{1}{b} & =\frac{\omega^{2}}{a b}\left(\frac{1}{a}-\frac{1}{b}\right) \\
\omega & =\sqrt{a b} \\
& =\sqrt{1 \times 2}=\sqrt{2} \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

## General Aptitude (GA) Question (Compulsory)

## Q. 56 - Q. 60 carry one mark each.

MCQ 1.56 One of the parts ( $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ ) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?
I requested that he should be given the driving test today instead of tomorrow.
(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

SOL 1.56 Option (B) is correct.
MCQ 1.57 If $(1.001)^{1259}=3.52$ and $(1.001)^{2062}=7.85$, then $(1.001)^{3321}$
(A) 2.23
(B) 4.33
(C) 11.37
(D) 27.64

SOL 1.57 Option (D) is correct option.
Let

$$
1.001=x
$$

So in given data :

$$
\begin{aligned}
x^{1259} & =3.52 \\
x^{2062} & =7.85 \\
x^{3321} & =x^{1259+2062} \\
& =x^{1259} x^{2062} \\
& =3.52 \times 7.85 \\
& =27.64
\end{aligned}
$$

Again

MCQ 1.58 Choose the most appropriate alternate from the options given below to complete the following sentence :
If the tired soldier wanted to lie down, he. $\qquad$ the mattress out on the balcony.
(A) should take
(B) shall take
(C) should have taken
(D) will have taken

SOL 1.58 Option (C) is correct.
MCQ 1.59 Choose the most appropriate word from the options given below to complete the following sentence :
Give the seriousness of the situation that he had to face, his. $\qquad$ was impressive.
(A) beggary
(B) nomenclature
(C) jealousy
(D) nonchalance

SOL 1.59 Option (D) is correct.
MCQ 1.60 Which one of the following options is the closest in meaning to the word given below?

## Latitude

(A) Eligibility
(C) Coercion
gate
(B) Freedom
(D) Meticulousness

SOL 1.60 Option (B) is correct.

## Q. 61 - Q. 65 carry two marks each.

MCQ 1.61 $A$ and $B$ are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a conditions that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that days is
(A) $1 / 4$
(B) $1 / 16$
(C) $7 / 16$
(D) $9 / 16$

SOL 1.61 Option (C) is correct.
The graphical representation of their arriving time so that they met is given as below in the figure by shaded region.


So, the area of shaded region is given by

$$
\begin{aligned}
& \text { Area of } \square P Q R S-(\text { Area of } \triangle E F Q+\text { Area of } \triangle G S H) \\
& =60 \times 60-2\left(\frac{1}{2} \times 45 \times 45\right) \\
& =1575
\end{aligned}
$$

So, the required probability $=\frac{1575}{3600}=\frac{7}{16}$
MCQ 1.62 One of the legacies of the Roman legions was discipline. In the legious, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.
Which one of the following statements best sums up the meaning of the above passage ?
(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals
(C) Disciplines was the armies inheritance from their seniors
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

SOL 1.62 Option (A) is correct.
MCQ 1.63 Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money values of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is
(A) 5
(B) 6
(C) 9
(D) 10

SOL 1.63 Option (A) is correct.
Let no. of notes of Rs. 20 be $x$ and no. of notes of Rs. 10 be $y$.
Then from the given data.

$$
\begin{aligned}
x+y & =14 \\
20 x+10 y & =230
\end{aligned}
$$

Solving the above two equations we get

$$
x=9, y=5
$$

So, the no. of notes of Rs. 10 is 5 .
MCQ 1.64 There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is
(A) 2
(B) 3
(C) 4
(D) 8

SOL 1.64 Option (A) is correct.
We will categorize the 8 bags in three groups as :
(i) $A_{1} A_{2} A_{3}$,
(ii) $B_{1} B_{2} B_{3}$,
(iii) $C_{1} C_{2}$

Weighting will be done as bellow
$1^{\text {st }}$ weighting $\rightarrow A_{1} A_{2} A_{3}$ will be on one side of balance and $B_{1} B_{2} B_{3}$ on the other. It may have three results as described in the following cases.
Case 1 : $\quad A_{1} A_{2} A_{3}=B_{1} B_{2} B_{3}$
This results out that either $C_{1}$ or $C_{2}$ willheavier for which we will have to perform weighting again.
$2^{\text {nd }}$ weighting $\rightarrow C_{1}$ is kept on the one side and $C_{2}$ on the other.
if $\quad C_{1}>C_{2} \quad$ then $C_{1}$ is heavier. $C_{1}<C_{2} \quad$ then $C_{2}$ is heavier.
Case 2: $\quad A_{1} A_{2} A_{3}>B_{1} B_{2} B_{3}$
it means one of the $A_{1} A_{2} A_{3}$ will be heavier So we will perform next weighting as:
$2^{\text {nd }}$ weighting $\rightarrow A_{1}$ is kept on one side of the balance and $A_{2}$ on the other.
if $\quad A_{1}=A_{2} \quad$ it means $A_{3}$ will be heavier
$A_{1}>A_{2} \quad$ then $A_{1}$ will be heavier
$A_{1}<A_{2} \quad$ then $A_{2}$ will be heavier
Case 3 : $\quad A_{1} A_{2} A_{3}<B_{1} B_{2} B_{3}$
This time one of the $B_{1} B_{2} B_{3}$ will be heavier, So again as the above case weighting will be done.
$2^{\text {nd }}$ weighting $\rightarrow B_{1}$ is kept one side and $B_{2}$ on the other
if $\quad B_{1}=B_{2} \quad B_{3}$ will be heavier
$B_{1}>B_{2} \quad B_{1}$ will be heavier
$B_{1}<B_{2} \quad B_{2}$ will be heavier
So, as described above, in all the three cases weighting is done only two times to give out the result so minimum no. of weighting required $=2$.

MCQ 1.65 The data given in the following table summarizes the monthly budget of an average household.

| Category | Amount (Rs.) |
| :--- | :--- |
| Food | 4000 |
| Clothing | 1200 |
| Rent | 2000 |
| Savings | 1500 |
| Other expenses | 1800 |

The approximate percentages of the monthly budget NOT spent on savings is
(A) $10 \%$
(B) $14 \%$
(C) $81 \%$
(D) $86 \%$

SOL 1.65 Option (D) is correct.

$$
\begin{aligned}
\text { Total budget } & =4000+1200+2000+1500+1800 \\
& =10,500
\end{aligned}
$$

The amount spent on saving $=1500$
So, the amount not spent on saving

$$
=10,500-1500=9000
$$

So, percentage of the amount

$$
\begin{aligned}
& =\frac{9000}{10500} \times 100 \% \\
& =86 \%
\end{aligned}
$$

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